

Signals & Systems

Laboratory no. 5 – Time-frequency signal analysis

Wavelet transform in Matlab

The basic instruction for setting coefficients of continuous wavelet transform is a function *cwt*.

$COEFS = cwt(S, SCALES, 'wname')$

where:

S – analyzed signal

Scales – scale parameter or scale vector.

Wname – name of the wavelet function

The list of available wavelet functions are summarized in the table below

Wavelet Family Short Name	Wavelet Family Name
'haar'	Haar wavelet
'db'	Daubechies wavelets
'sym'	Symlets
'coif'	Coiflets
'bior'	Biorthogonal wavelets
'rbio'	Reverse biorthogonal wavelets
'meyr'	Meyer wavelet
'dmey'	Discrete approximation of Meyer wavelet
'gaus'	Gaussian wavelets
'mexh'	Mexican hat wavelet
'morl'	Morlet wavelet
'cgau'	Complex Gaussian wavelets
'shan'	Shannon wavelets
'fbsp'	Frequency B-Spline wavelets
'cmor'	Complex Morlet wavelets

To determine the relationship between the scale parameter and the frequency, use the command:

```
F = scal2frq(A,'wname',DELTA)
```

where :

A – is a scale parameter

Wname – name of the wavelet function

DELTA – is a sampling period

EXAMPLE

1. Generate a signal consisting of a sum of three sine waves (e.g. 3, 5 and 10 Hz).
2. With use of the *cwt* plot the scalogram of the signal.

e.g. *f = cwt(signal, scale_vector, 'wavelet_function', 'plot')*

Hint:

Scale vector should be adjusted so as to cover the entire frequency band of the signal (using the function *scal2frq*). For example, if the sampling frequency is 100 Hz then:

```
scal2frq(5,'cmor 1-1',1/100)
```

```
ans =
```

```
20
```

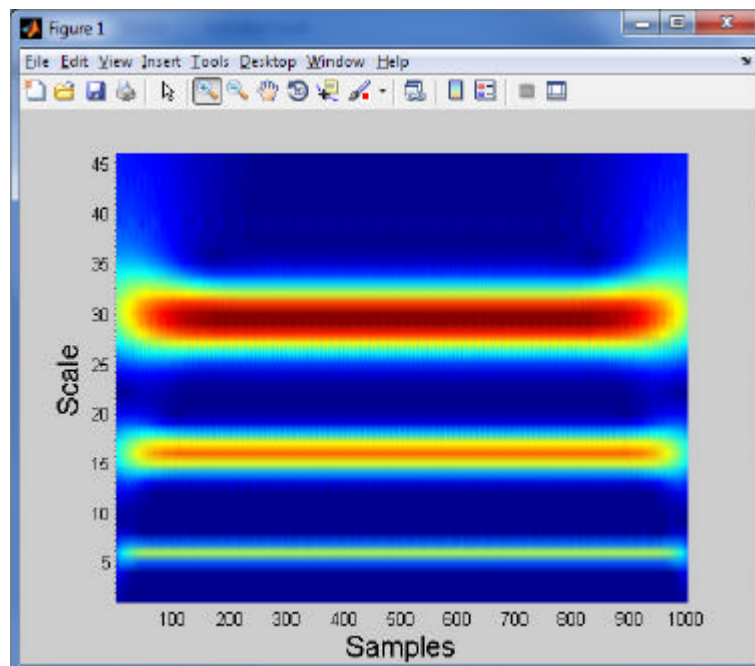
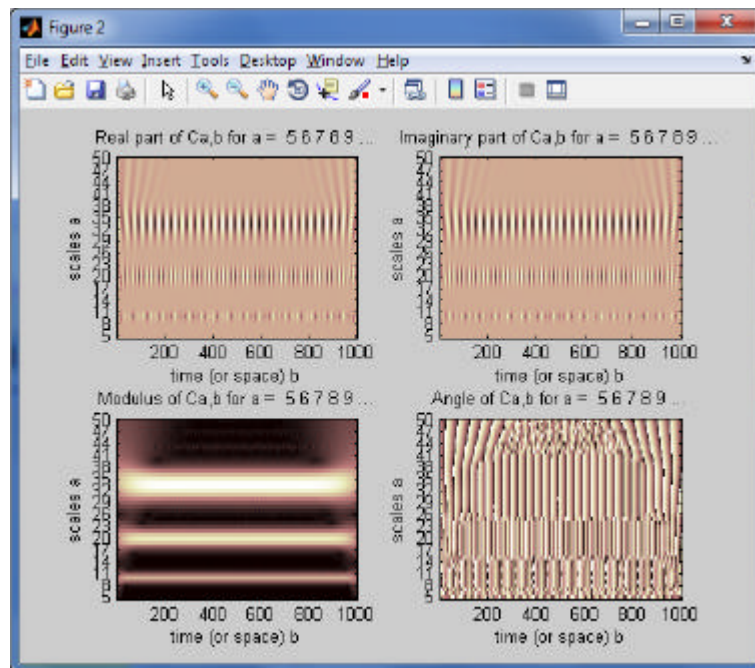
```
scal2frq(50,'cmor 1-1',1/100)
```

```
ans =
```

```
2
```

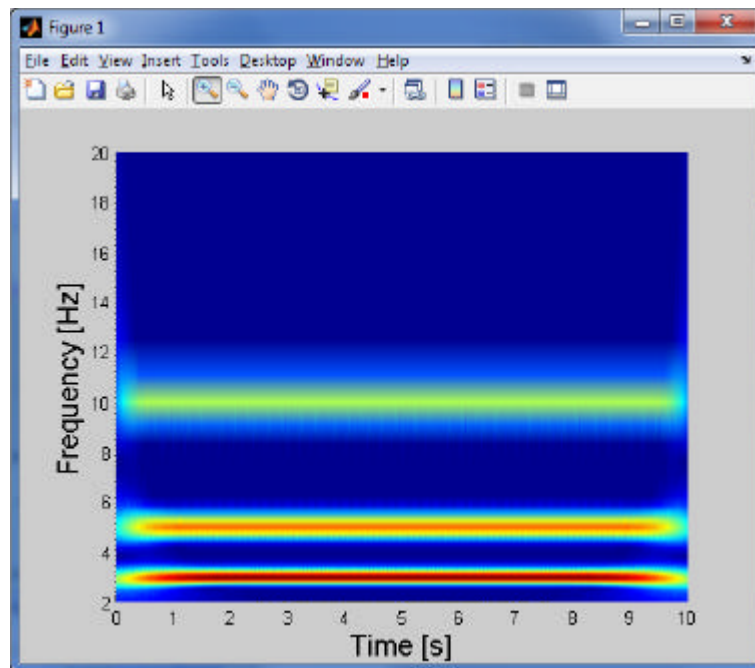
In order to analyze the signal in the range of 2 to 20 Hz scale vector must be such as [5:1:50]; The last parameter of the *cwt* command will generate the scalegram separately for the real and complex part, and the absolute value and phase of wavelet coefficients (Fig. 2). Scalegram can also be plotted using *surf* function (Fig. 1), e.g.

```
surf(abs(f))  
shading interp  
view([0 90])  
axis Wight  
ylabel('Scale','FontSize',16)  
xlabel('Samples','FontSize',16)
```



Note that in this case scalegram the x-axis is calibrated in the samples, and the y axis in scale parameters. To rescale the axes the following way can be used:

```
time = repmat(t,46,1)
fr = scal2frq(5:1:50,'cmor 1-1',1/100);
freq = repmat(fr,1001,1);
surf(time,freq',abs(f))
shading interp
view([0 90])
```



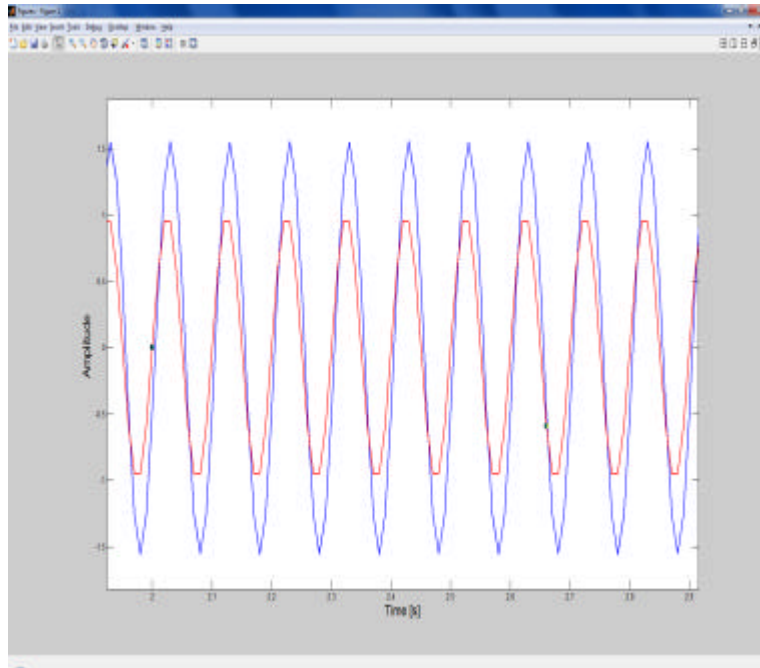
Signal reconstruction

For such an analysis, it is possible to partially reconstruct the individual components of the signal

EXAMPLE

In order to reconstruct the individual components of the signal we perform the analysis of the scalegram. Similarly to the frequency analysis, we determine the maxima of the given scale parameter. For example, as described above, we obtain the maximum values for the scale parameters 6 (+4), 16 (4) and 29 (+4) (Note that the taken scale vector, began with 5 and the scale of the figure is a relative scale – not relating to the size of the matrix of wavelet coefficients). For setting the time histories from the matrix of coefficients, we can use following code

```
plot(t,real(f(6,:)))
hold on
plot(t,y3,'r')
```

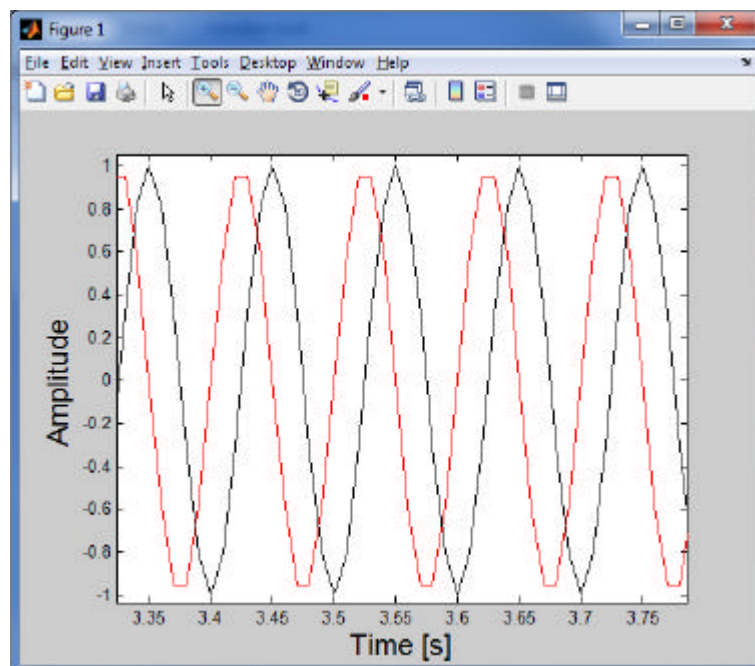


As one can see the time histories are different in both amplitude and phase. In order to further reconstruct the signal it is necessary to apply the formula:

$$x(t) = C_{lg}^{-1} \int W(a, b) \frac{da}{a^2}$$

```
s2=detrend(cumsum(real(f(scale,:))/scale^(3/2)));
s2a=detrend(s2);
res_sig=(0.5/(2*pi*fc))^( -1)*s2a;
```

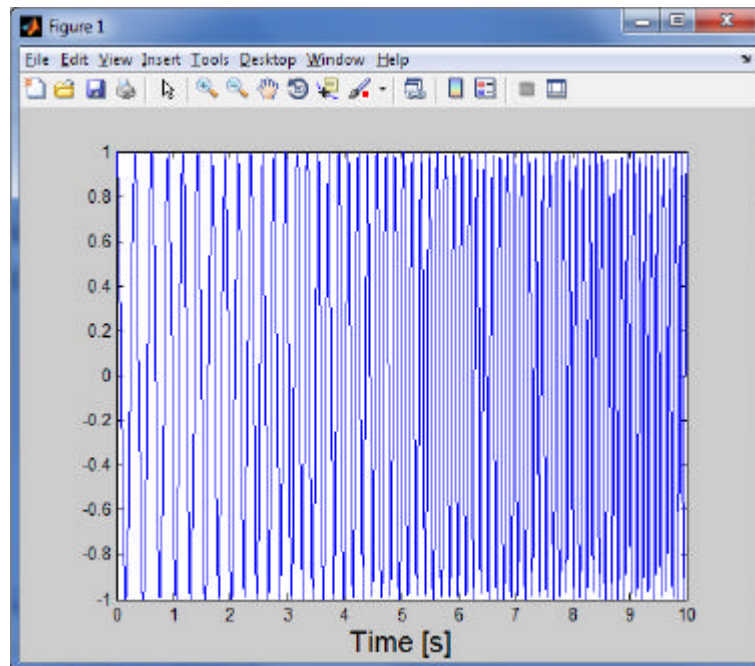
In this case, the time histories are scaled to the amplitude.



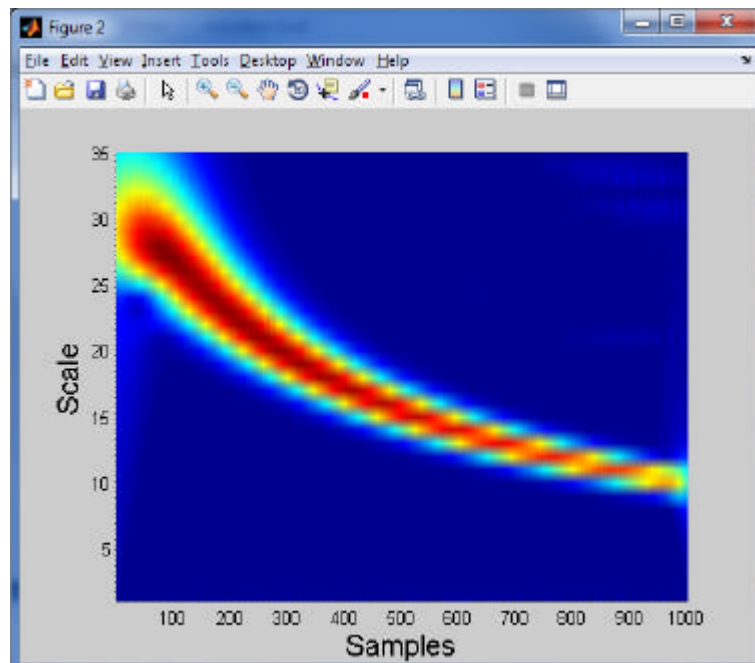
Analysis of non-stationary signals

To perform this analysis we first generate the non-stationary signal e.g. sine wave with linearly change of frequency:

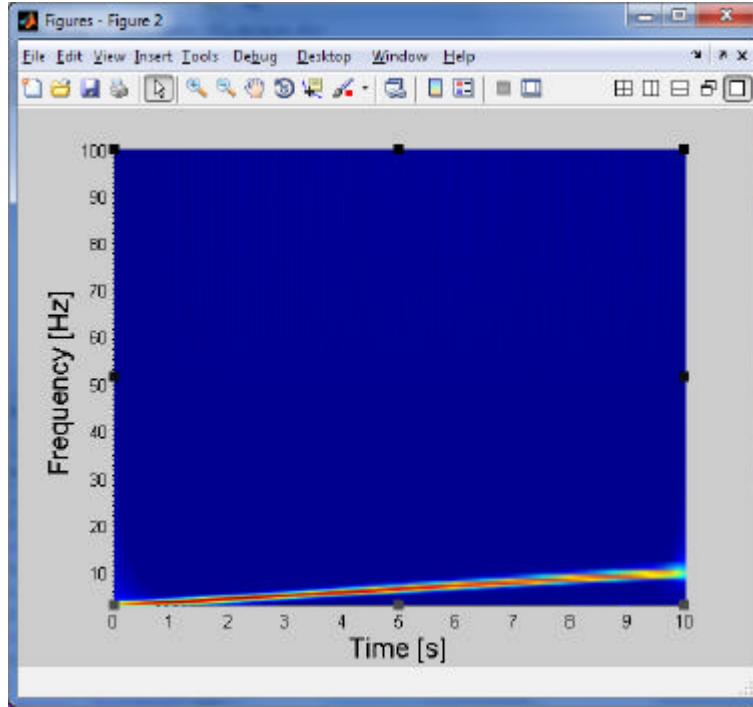
```
y4 = chirp(t,3,10,10);
```



For such a signal determine scalegram using continuous wavelet transform.



Rescale the plot (time - frequency)



Identification of a system mechanical parameters

In order to present the method the response of the n DOF system was assumed as expressed by the following relationship:

$$x(t) = \sum_{j=1}^n A_j e^{-\alpha_j t} \sin(\omega_{dj} t),$$

Where: A_j amplitude of j – th mode shape,

α_j modal damping factor,

ω_{nj} undamped natural frequency,

ω_{dj} natural frequency.

The wavelet transform of the signal $x(t)$

$$W_g x(a, b) = \sqrt{a} \sum_{j=1}^n A_j e^{-\alpha_j b} e^{-\frac{(a\omega_{dj} - \omega_0)^2}{2}} e^{j\omega_{dj} b},$$

where element $e^{-\frac{(a\omega_{dj} - \omega_0)^2}{2}}$ in the above equation is the derivative from the Fourier transform of the Morlet wavelet for the given scale parameter. This means that the component is the Gaussian window in the frequency domain, acting as a band-pass filter. For a fixed value of the dilation parameter, as the skeleton of wavelet transform we obtain the signal component whose frequency is:

$$\mathbf{w}_{dj} = \frac{\mathbf{w}_0}{a_j}.$$

assuming that:

$$e^{-\left(a\mathbf{w}_{dj}-\mathbf{w}_0\right)^2} \cong 1,$$

One can write:

$$W_g x(a_j, b) = \sqrt{a_j} A_j e^{-\mathbf{x}_j \mathbf{w}_{nj} b} e^{-j \mathbf{w}_{dj} b}.$$

Using this property it is possible to trace the signal envelope and assess the modal parameters for each mode independently.

After further mathematical transformations, one notes that for i – th scale parameter the following dependence is satisfied:

$$\ln \left(\frac{|W_g x(a_i, b)|}{\sqrt{a_i}} \right) = \mathbf{x}_i \mathbf{w}_{ni} b + \ln(A_i).$$

It follows that the modal damping factor of the system can be designated as the slope coefficient obtained from the logarithm of the module of wavelet coefficients for the given scale parameter a_i .

$$\zeta = \frac{\ln(W_g x(a, b))}{\omega_n}$$

Determination of the system natural frequencies consists of the detection of the wavelet transform back. The most common method of its detection is to seek maximum value of wavelet coefficients in the matrix of wavelet coefficients

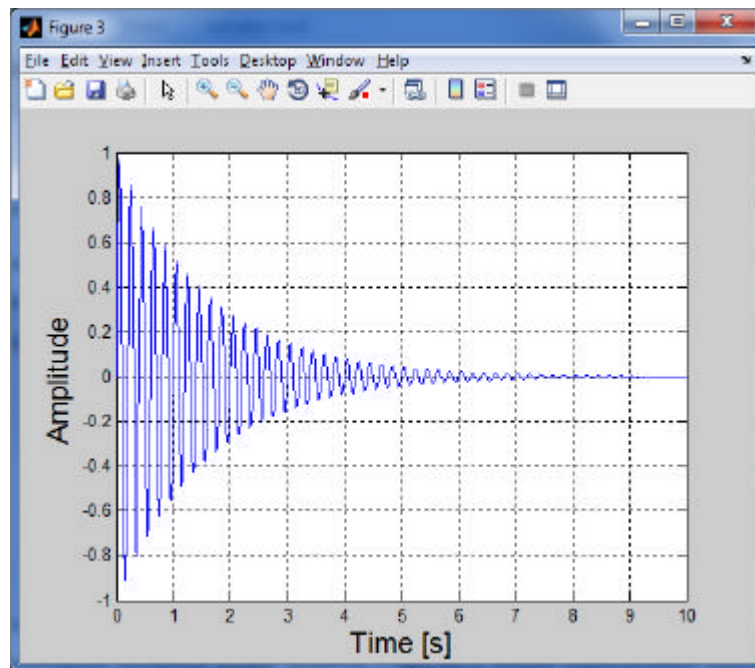
EXAMPLE

Generate a signal of a single degree of freedom system, described by the equation:

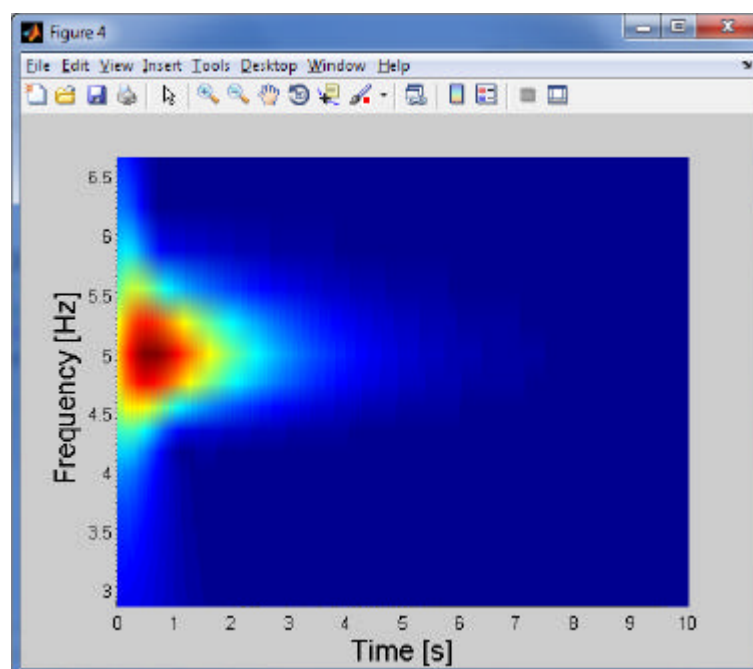
$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

e.g.:

$$e = \exp(-0.02 * 2 * \pi * 5 * t) .* \sin(2 * \pi * 5 * t);$$

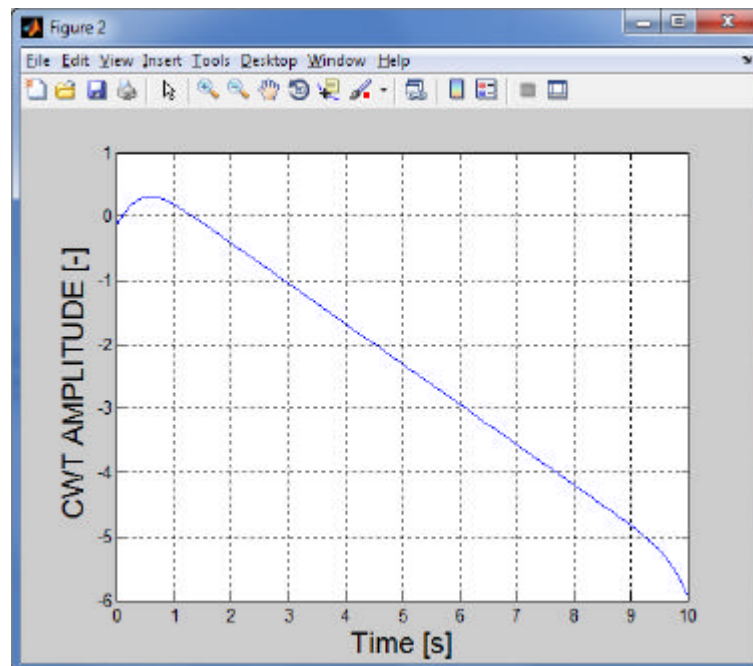


Plot the scalegram of the system

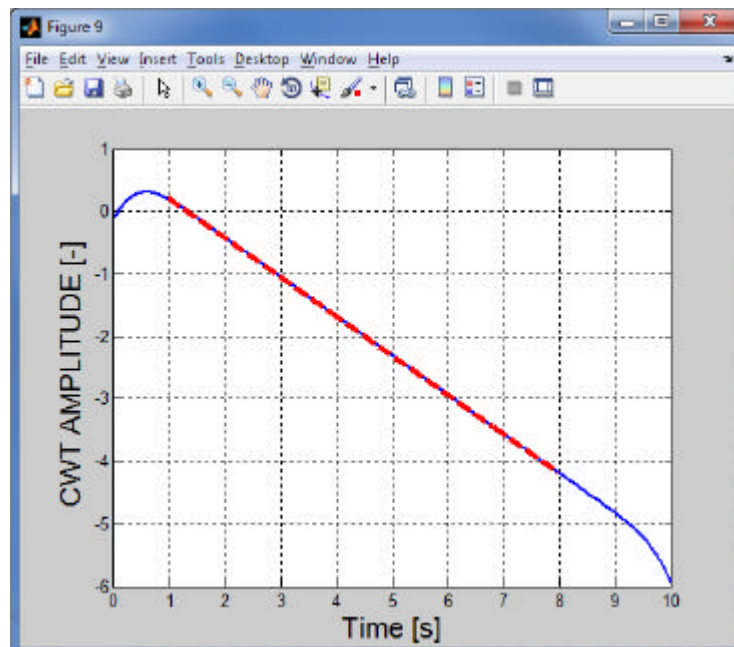


Calculate the logarithm of the reconstructed signal envelope (for the scale parameter corresponding to the generated signal frequency).

```
l=log(abs(f(20,:)));
```



Using a function `polyfit` fit the straight line to the given time history (in its linear range).



Exercise 1.

With use of wavelet transform and example given above identify parameters of the 2 DOF harmonic oscillator, from its impulse response. Pay attention to proper selection of the analysis parameters (sampling frequency, scale parameters ect.)

Exercise 2.

Generate a signal with a frequency varying in time in accordance with the freely chosen relationship (linear, quadratic, logarithmic, etc.). Build sklaogram of such a signal. Based on the analysis of the matrix of wavelet coefficients (maxima), set a change of frequency as a function of time.