# Signals & Systems

# Laboratory no. 8 – Prediction error method

The least squares method, which was the subject of previous laboratory is certainly easy to use, although to obtain converged estimators requires a relatively strong assumptions. In some cases, non- convergence, however, can be tolerated. For example, if the signal to noise ratio is high, this bias will be small. When designing the controller, some bias of the model estimator is allowed, because properly designed control system has a certain robustness to changes of the model parameters. There are situations in which converged estimators are of great importance. The least squares method can be modified in the way to give converged estimators. These modifications are:

1. Minimizing prediction errors for more complex structures of parametric models. This leads to the prediction error methods of the class, which will be dealt with in this chapter
2. Modification of the normal equations corresponding to the least squares method estimator. This further leads to an instrumental variable methods.

In this laboratory exercises we will deal with the prediction error method. Its schedule can be described as follows:

1. Select the structure of the model:

$$\mathbf{y}(t) = \mathbf{G}(q^{-1};q)\mathbf{u}(t) + \mathbf{?}\ (q^{-1};q)\mathbf{e}(t)$$

and predictor

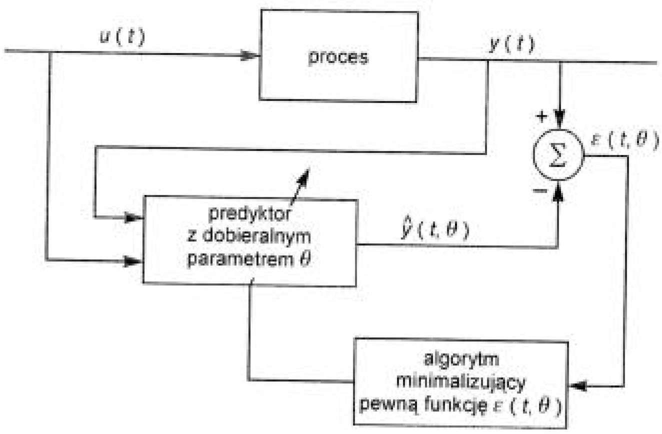
$$\hat{\mathbf{y}}(t\,|\,t-1;q) = L_1(q^{-1};q)y(t) + L_2(q^{-1};q)u(t)$$

which depends only on past data. Filters $L_1(0;q)=0$ and $L_2(0;q)=0$

2. Select the objective function $h(Q)$

3. Set assessment of the parameters $q$ as a global point that minimizes the loss function $h(R_N(q))$

Prediction error method diagram is shown below:

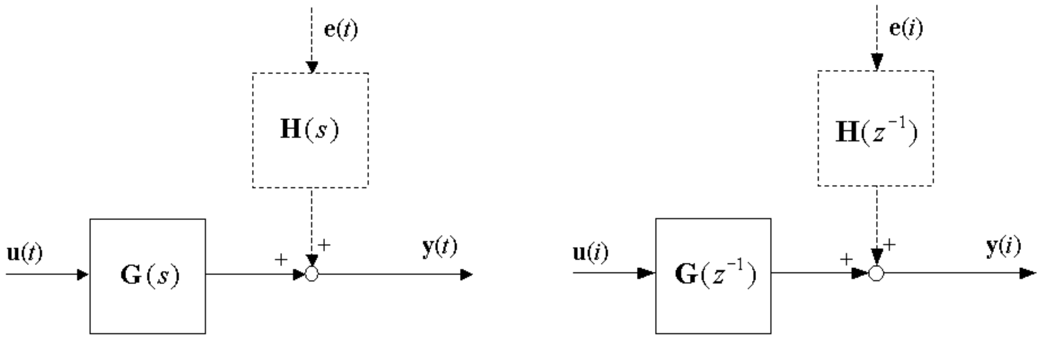


Regression models the most often use the operator definition of system dynamics in form of transmittance (transfer function), or state space equations. The input-output and disturbance-output transmittances were selected to model the system. It means that for the polynomial models the disturbance is reduced the output of the object

$$\mathbf{y}(t) = \mathbf{G}(s)\mathbf{u}(t) + ?\ (s)\mathbf{e}(t) \tag{1}$$

or

$$\mathbf{y}(i) = \mathbf{G}(z^{-1})\mathbf{u}(i) + ?\ (z^{-1})\mathbf{e}(i) \tag{2}$$




Transmittances of objects can be defined by the following polynomials:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} +, \ldots, + a_{nA} z^{-nA}, \tag{3}$$

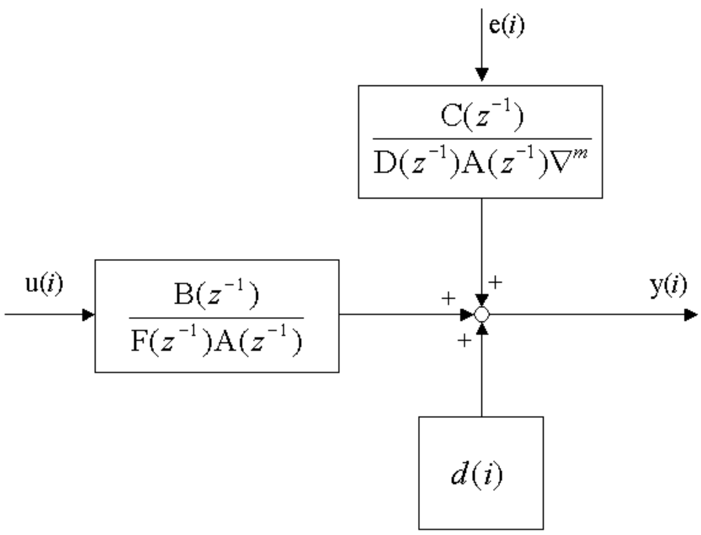
$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} +, \ldots, + b_{nB} z^{-nB}, \tag{4}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} +, \ldots, + c_{nC} z^{-nC}, \tag{5}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} +, \ldots, + d_{nD} z^{-nD}, \tag{6}$$

$$F(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-2} +, \ldots, + f_{nF} z^{-nF}. \tag{7}$$

Taking into account the impact of all possible disturbances, generalized model of a system is located below.



Polynomial general SISO model is defined as follows after substituting in (2) the individual polynomials:

$$A(z^{-1})\left[\nabla^m y(z^{-1})\right]=\frac{B(z^{-1})}{F(z^{-1})}\left[\nabla^m u(z^{-1})\right]+\frac{C(z^{-1})}{D(z^{-1})}e(z^{-1}) \quad (8)$$

In order to build a dynamic model of the system the future structure of the model should be determined (Table 1). We do this by including the selected polynomials.

Table 1. Overview of selected parametric regression models

| Nazwa modelu | Użyte wielomiany | Transmitancja wejście-wyjście | Transmitancja zakłócenia-wyjście | Wektor zmiennych $\varphi(i)$ oraz wektor parametrów $\theta$ |
|---|---|---|---|---|
| **ARX** | A, B | $K(z^{-1}) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}$ | $H(z^{-1}) = \frac{1}{A(z^{-1})}$ | $\varphi(i) = [y(i-1) \ \dots \ y(i-dA) \ \ u(i-d) \ \dots \ u(i-d-dB)]^T$<br>$\theta = [a_1 \ \dots \ a_{dA} \ \ b_0 \ \ b_1 \ \dots \ b_{dB}]^T$ |
| **ARMAX** | A, B, C | $K(z^{-1}) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})}$ | $H(z^{-1}) = \frac{C(z^{-1})}{A(z^{-1})}$ | $\varphi(i) = [y(i-1) \ \dots \ y(i-dA) \ \ u(i-d) \ \dots \ u(i-d-dB)$<br>$e(i-1) \ \dots \ e(i-dC)]$<br>$\theta = [a_1 \ \dots \ a_{dA} \ \ b_0 \ \ b_1 \ \dots \ b_{dB} \ \ c_1 \ \dots \ c_{dC}]^T$ |
| **FIR** | B | $K(z^{-1}) = z^{-d}B(z^{-1})$ | $H(z^{-1}) = 1$ | $\varphi(i) = [u(i-d) \ \dots \ u(i-d-dB)]^T$<br>$\theta = [b_0 \ \ b_1 \ \dots \ b_{dB}]^T$ |
| **OE** | B, F | $K(z^{-1}) = z^{-d}\frac{B(z^{-1})}{F(z^{-1})}$ | $H(z^{-1}) = 1$ | $\varphi(i) = [\hat{y}(i-1 \mid \theta) \ \dots \ \hat{y}(i-dF \mid \theta) \ \ u(i-d) \ \dots$<br>$\dots \ u(i-d-dB)]$<br>$\theta = [f_1 \ \dots \ f_{dF} \ \ b_0 \ \ b_1 \ \dots \ b_{dB}]^T$ |
| **BJ** | B, C, D, F | $K(z^{-1}) = z^{-d}\frac{B(z^{-1})}{F(z^{-1})}$ | $H(z^{-1}) = \frac{C(z^{-1})}{D(z^{-1})}$ | $\theta = [f_1 \ \dots \ f_{dF} \ \ b_0 \ \ b_1 \ \dots \ b_{dB} \ c_1 \ \dots \ c_{dC} \ d_1 \ \dots \ d_{dD}]^T$ |
| **PEM** | A, B, C, D, F | $K(z^{-1}) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})F(z^{-1})}$ | $H(z^{-1}) = \frac{C(z^{-1})}{A(z^{-1})D(z^{-1})}$ | $\theta = [f_1 \ \dots \ f_{dF} \ \ b_0 \ \ b_1 \ \dots \ b_{dB} \ c_1 \ \dots \ c_{dC} \ d_1 \ \dots \ d_{dD}$<br>$a_1 \ \dots \ a_{dA}]^T$ |
| **SSIF** | A', B', C' | $x(i+1) = A'x(i) + B'u(i) + w(i)$<br>$y(i) = C'x(i) + v(i)$<br>oraz istnieje związek<br>$K(z^{-1}) = C'[zI - A']^{-1}B'$ | | $\varphi(t) = [\hat{x}^T(t \mid \theta) \ \ u^T(t) \ \ \varepsilon^T(t \mid \theta)]$ |

**EXAMPLE**

The object with transmittance was identified:

$$H(s) = \frac{1}{s^2 + 10s + (2p\,30)^2} + \frac{1}{s^2 + 2s + (2p\,80)^2}$$

Its output is transformer by the nonlinear static function $f_N$

$$f_N(.) = f_N\left[10^9 \cdot y(t)^3\right]$$

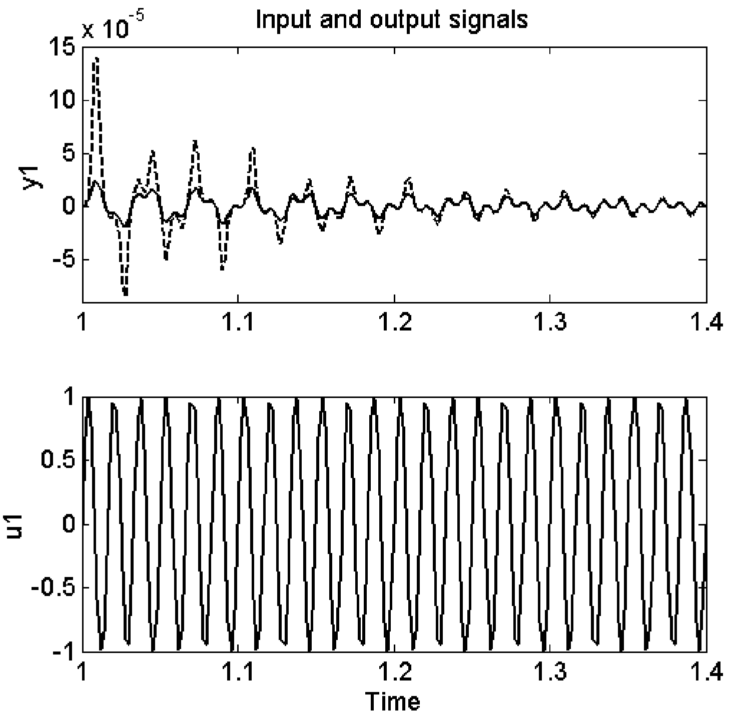
$$y(s) = f_N\left[\frac{2s^2 + 12s + 288200}{s^4 + 12s^3 + 288200s^2 + 2598000s + 8.977 \cdot 10^8} u(s)\right]$$

The object was excited by the harmonic signal in form:

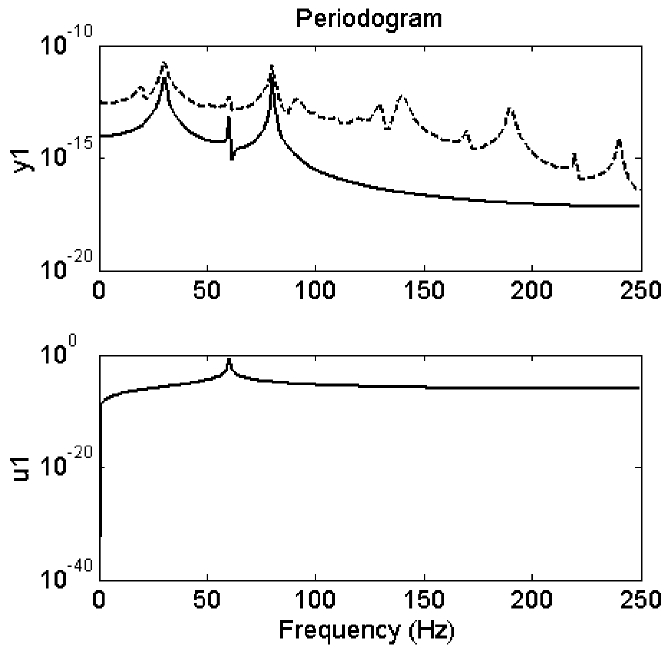
$$u(t) = \sin(\,2p\,60t)$$

where t∈(0,0.8) [s], sampling frequency $f_p$=500 [Hz]

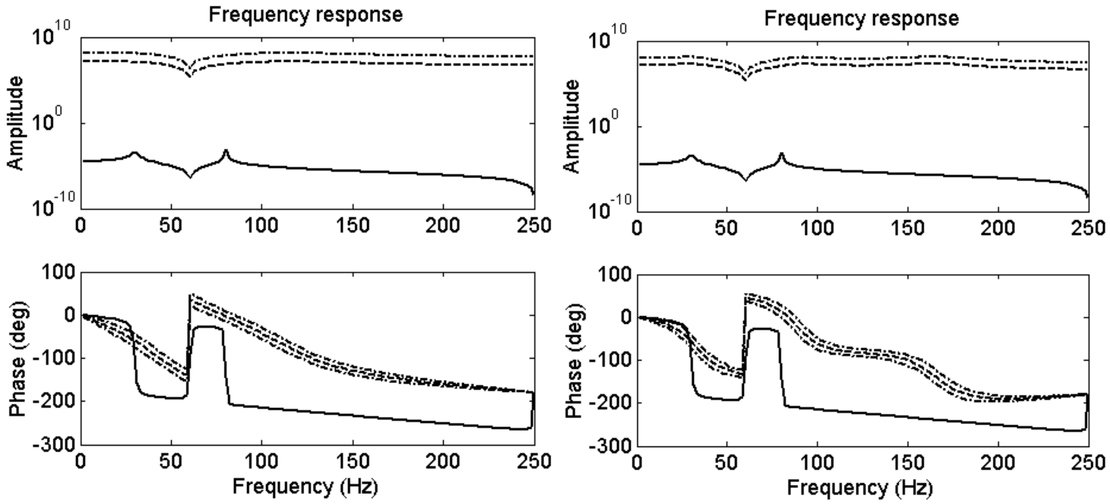
$$y(z) = f_N\left[\frac{3.798 \cdot 10^{-6} z^3 - 1.695 \cdot 10^{-6} z^2 - 1.705 \cdot 10^{-6} z + 3.738 \cdot 10^{-6}}{z^4 - 2.911z^3 + 3.945z^2 - 2.882z + 0.9763} u(z)\right]$$



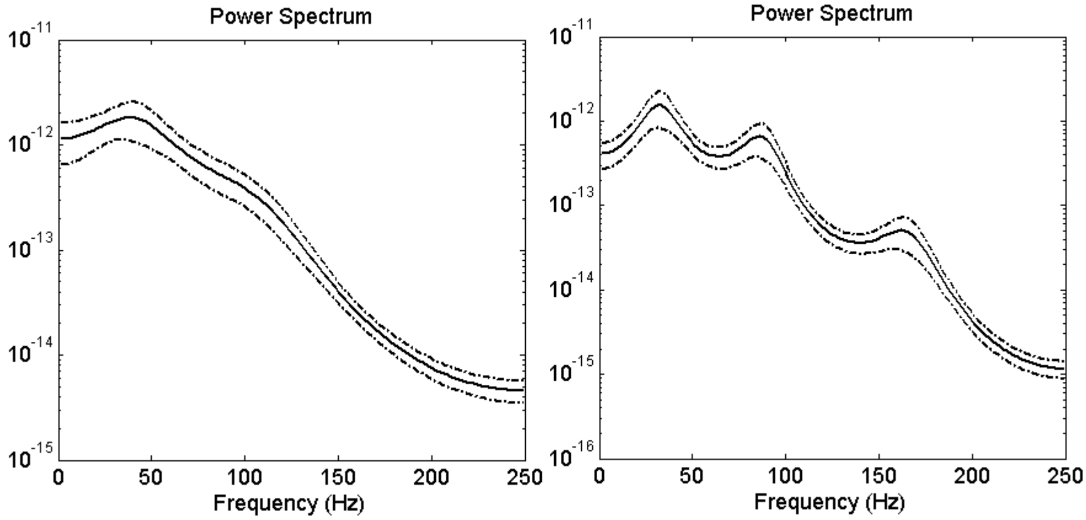

Time histories of the input and output signals of the simulated object: linear case (solid line) and nonlinear case (dashed)



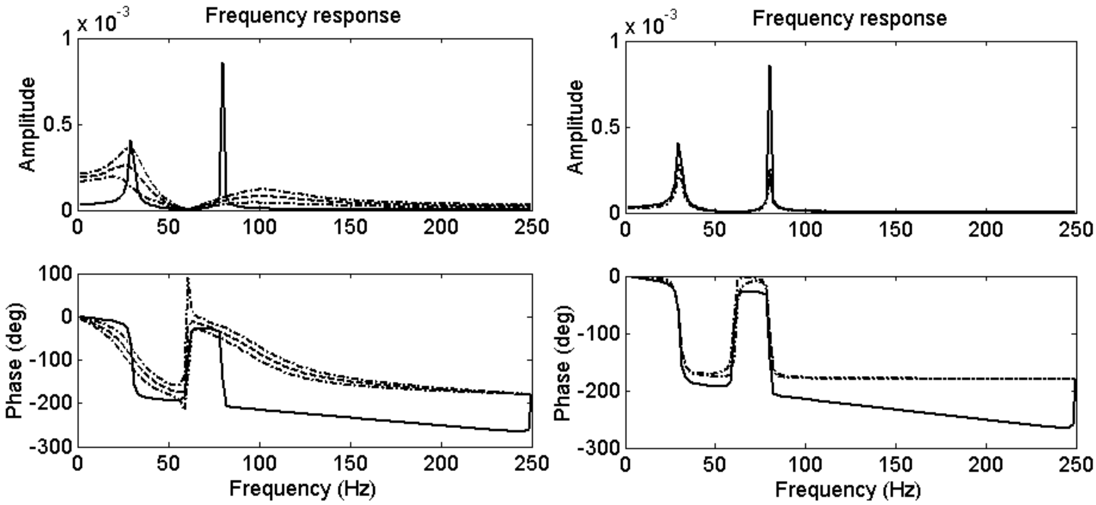

Power spectra in logarithmic scale of the input and output signals of the simulated object: linear case (solid line) and nonlinear case (dashed)



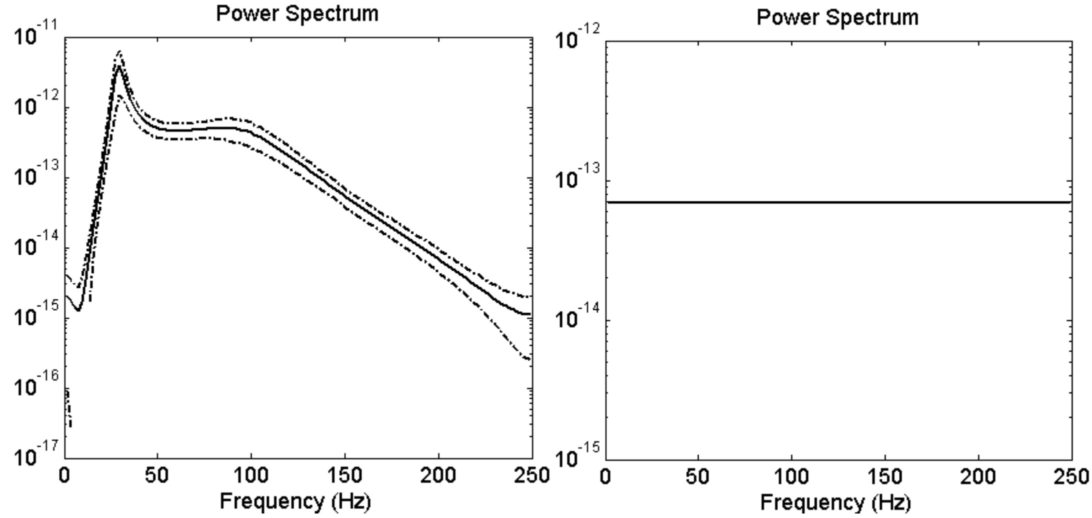

Examples of identification of nonlinear simulated object: model ARX (4,3,1) (left side) and the model ARX (6,3,1) (right side). Solid line means the discrete model, the dashed line the - ARX model, the dash-dot line - confidence interval at level 1- $a$ = 0.95, the amplitude in a logarithmic scale



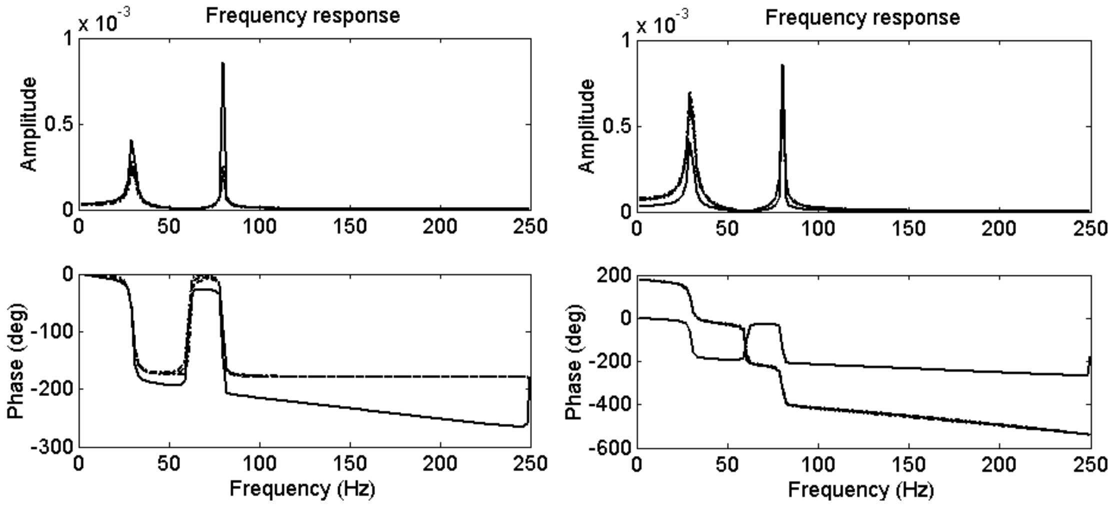

Examples of identification of nonlinear simulated object: model ARX (4,3,1) (left side) and the model ARX (6,3,1) (right side) for the disturbance transmittance $H(z^{-1})$.. Solid line means the discrete model, the dash-dot line - confidence interval at level $1- a = 0.95$, the amplitude in a logarithmic scale



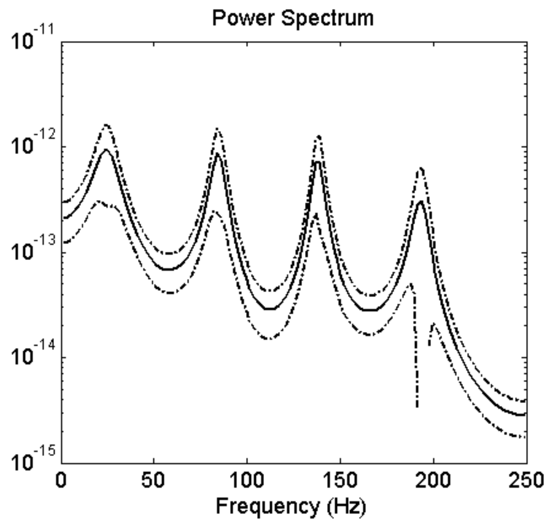

Examples of identification of nonlinear simulated object: model ARMAX (4,3,3,1) (left side) and the model OE (3,4,1) (right side). Solid line means the discrete model, the dashed line the – ARMAX/OE model, the dash-dot line - confidence interval at level $1- a = 0.95$, the amplitude in a linear scale



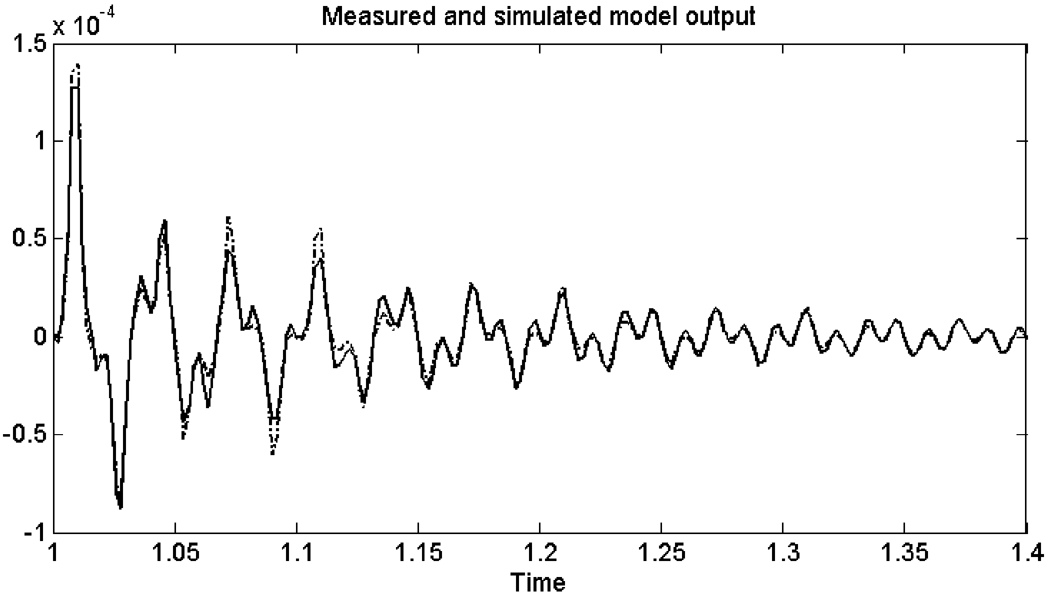

Examples of identification of nonlinear simulated object: model ARMAX (4,3,3,1) (left side) and the model OE (3,4,1) (right side) for the disturbance transmittance $H(z^{-1})$.. Solid line means the discrete model, the dash-dot line - confidence interval at level 1- $a$ = 0.95, the amplitude in a logarithmic scale



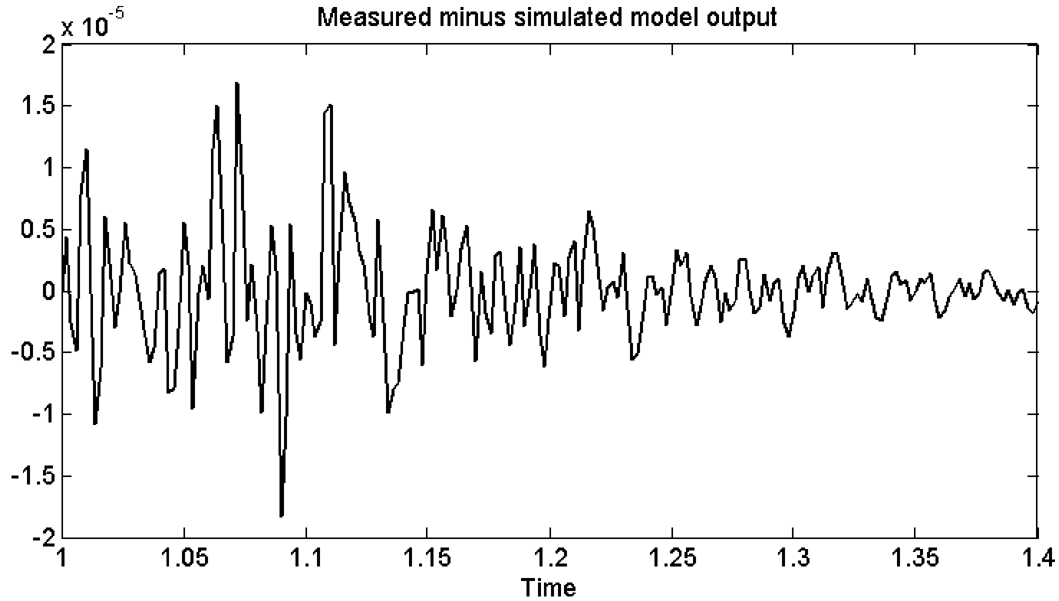

Examples of identification of nonlinear simulated object: model BJ (3,2,8,4,1) (left side) and the model BJ (4,2,8,4,1) (right side). Solid line means the discrete model, the dashed line the – BJ model, the dash-dot line - confidence interval at level 1- $a$ = 0.95, the amplitude in a linear scale




Examples of identification of nonlinear simulated object: model BJ (3,2,8,4,1) (left side) and the model BJ (4,2,8,4,1) (right side) for the disturbance transmittance $H(z^{-1})$ (identical polynomials orders in the disturbance transmittance). Solid line means the discrete model, the dash-dot line - confidence interval at level 1- $a$ = 0.95, the amplitude in a logarithmic scale



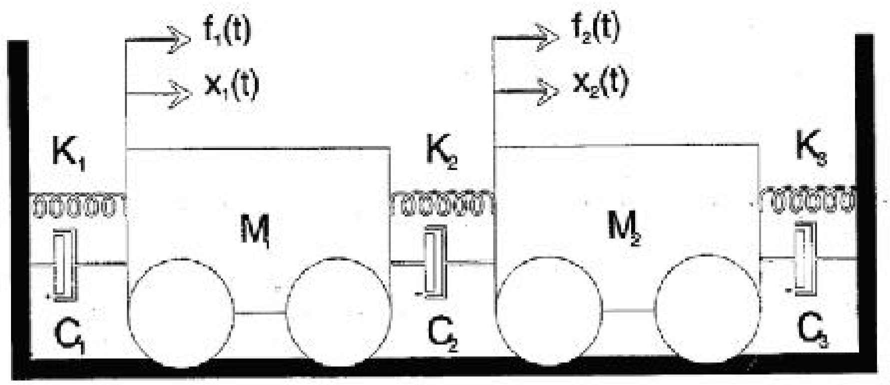

Examples of output simulation based on the identified model of the nonlinear object – BJ (3,2,8,4,1). Dash-dot line - confidence interval at level 1 - ? = 0.95, the amplitude in a logarithmic scale




Examples of output simulation error based on the identified model of the nonlinear object – BJ (3,2,8,4,1). Dash-dot line - confidence interval at level 1 - ? = 0.95, the amplitude in a logarithmic scale

Exercise 1.

Model in Simulink the system from the figure below and simulate it by forcing it with random noise. Time histories of both input (force $f_1$) and output (displacement $x_2$) save to the Matlab workspace




Data: $M_1 = 10$, $M_2 = 7$, $C_1 = 0.7$, $C_2 = 0.9$, $C_3 = 0.7$, $K_1 = 120$, $K_2 = 200$, $K_3 = 170$

Exercise 2.

For the data saved in Exercise 1 identify the parameters of the models of class ARMAX, BJ, OE and PEM. Take comparable orders of the polynomials and delay equal 0.

## Exercise 3.

Simulate the identified models with use of the input signal $f_1$ (stored in Exercise 1). Compare obtained results.

## Exercise 4.

Repeat Exercises 2 and 3 adding earlier the 0.1 noise to the data.