FFT and Spectral Leakage

Introduction:

Frequency Transform is used to study a signal's frequency domain characteristics. When using FFT to study the frequency domain characteristics of a signal, there are two limits : 1) The detectability of a small signal in the presence of a larger one ; 2) frequency resolution - which distinguishes two different frequencies.

In practice, the measured signals are limited in time and the FFT calculates the frequency transform over a certain number of discreet frequencies called bins.

Spectral Leakage:

In reality, signals are of time-limited nature and nothing can be known about the signal beyond the measured interval. For example, if the measurement of a never ending continuous train of sinusoidal wave is of interest, at some point of time we need to terminate our measurement to do further analysis. The limit on the time is also posed by limitations of the measurement system itself (like buffer size) besides other factors. Some assumptions have to made about the nature of the signal outside the measured interval. Fourier Transforms implicitly assumes that the signal essentially repeats itself after the measured interval.

Figure 1 illustrates the scenario in which a continuous train of sinusoidal signal is observed over a finite interval of time ("measured signal"). As discussed, the FFT assumes the signal to be continuous (conceptually, it does this by juxtaposing the measured signal repetitively). Observe the glitches in the assumed signal. These glitches are the manifestations of the measurement time relative to the frequency of the actual signal. If measurement time is an integral multiple of the rate of the actual signal (i.e. the inverse of the frequency of the signal), then no glitch will be observed in the assumed signal. In Figure 1, the measurement time is purposefully made to be a non-integral multiple of the actual signal rate. These sharp discontinuities will spread out in the frequency domain. This is called spectral leakage.

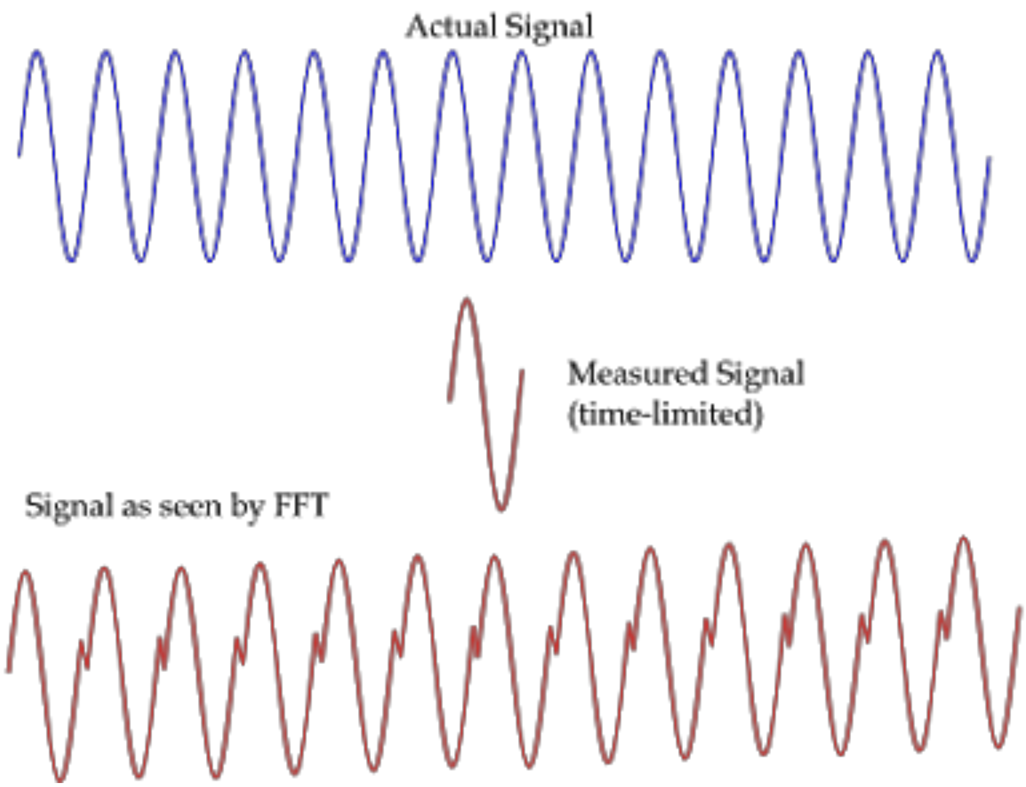

Figure 1: Impact of observation interval on FFT

Lets visualize the concept of spectral leakage by taking a pure sinusoidal signal as an example. Here we consider a sinusoid of 7Hz frequency (7 cycles in 1 second) and sampling it with a sampling frequency Fs=100Hz. We observe the signal for 100 seconds (700 cycles in total) and take the FFT of the observed signal. Figure 2 illustrates the frequency spectrum of the observed/measured signal. Essentially, the frequency spectrum contains a distinct peak at 7Hz.

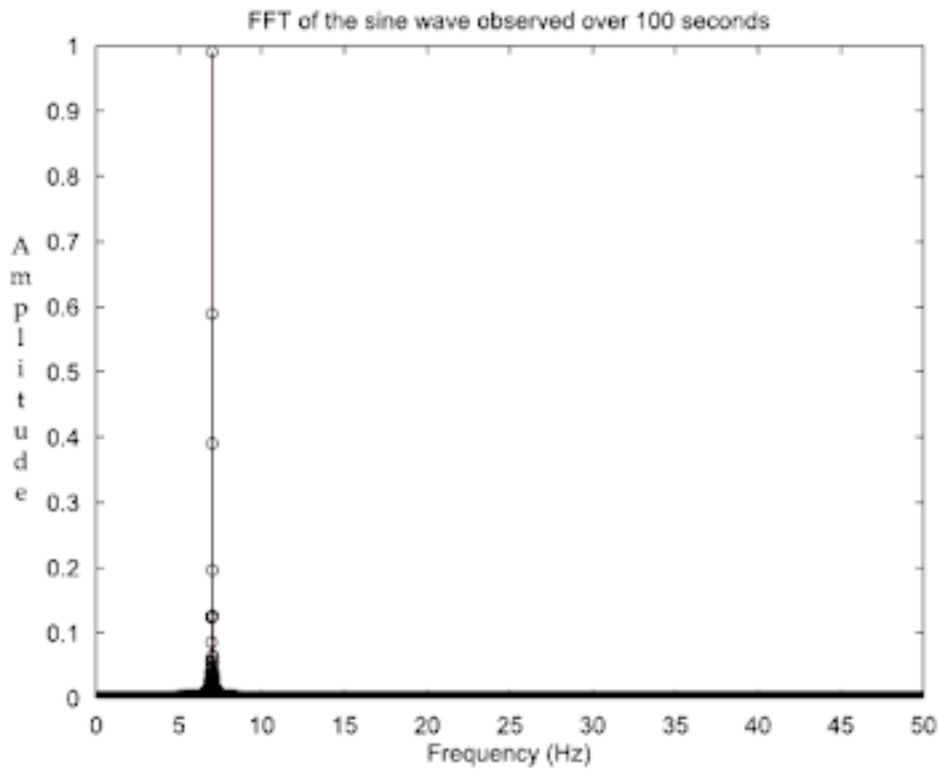

Figure 2: Frequency Spectrum of a 7 Hz sinusoid observed for 100 seconds

Next, the measurement window is shrunk to 1 second. Now the FFT is taken on this observed signal. Figure 3 illustrates the frequency spectrum of the observed signal (which has 7 cycles only). The frequency spectrum contains some spectral leakage because of limited observation interval.

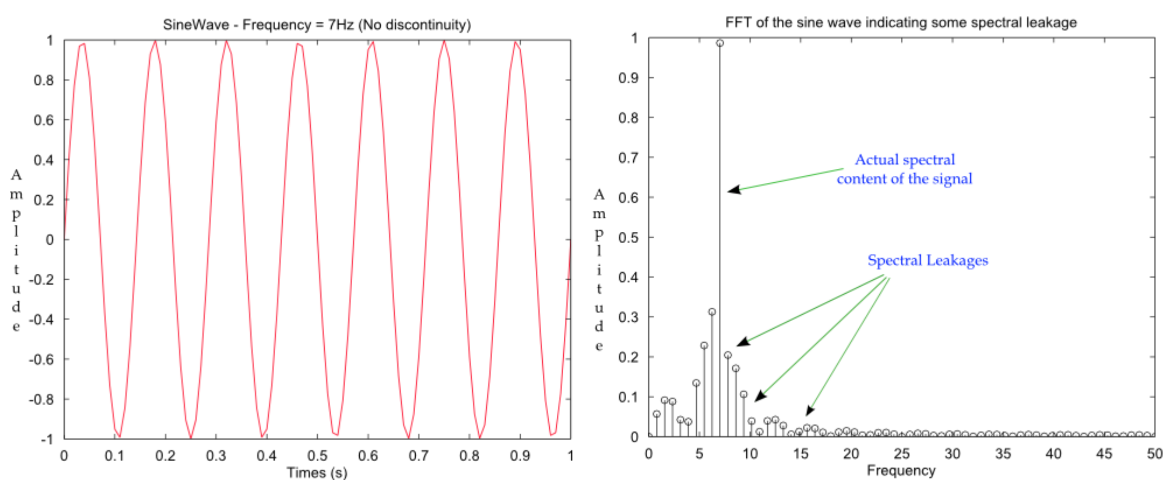

Figure 3 : A 7 Hz sinusoid observed for 1 second and its

Frequency Spectrum

In the previous illustrations (Figure 2 and 3), the observation time interval contained an integral number of sinewave cycles (i.e. in Figure 2 the observation interval contained exactly 700 cycles of sinusoid and in Figure 3 the observation interval contained exactly 7 cycles of sinusoid in the time domain).

For the next illustration the measurement time interval is adjusted in such a way that number of cycles in the observation window is no longer an integer. In Figure 4, the signal is observed for 1.4 seconds, which implies that there are 9.8 cycles. Now the observed signal does not end at zero amplitude at t=1.4 seconds. This scenario gives rise to gliches in the FFT's assumed signal (which constructs a periodic signal from the observed signal) and obviously results in more spectral leakage (Compare the Frequency spectrum in Figure 3 and Figure 4).

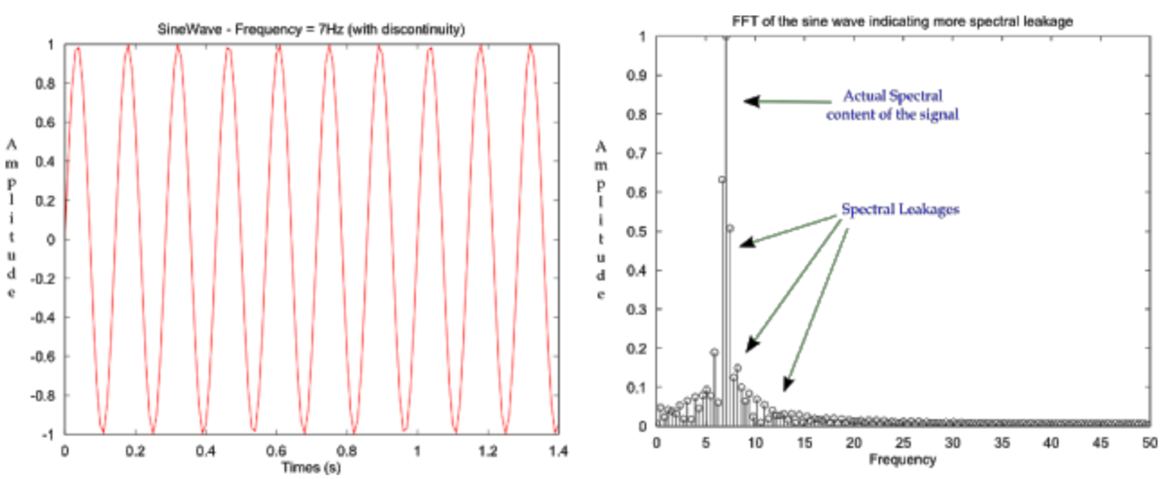

Figure 4 : A 7 Hz sinusoid observed for 1.4 seconds and its Frequency Spectrum.

The effect of spectral leakage may be lessened if the observed signal does not contain any discontinuity at the end of the measurement time (this scenario rarely occurs in any real application). Another scenario in which the spectral leakage can be reduced is by having a signal that gradually reduces to zero at the ends of the measurement time.(All the windows (like Hamming, Hanning, Bartlett, etc.., essentially attempts to do this). Such signal would have no discontinuity when made periodic and so does not suffer spectral leakages.

Conclusion:

Spectral leakage is not due to FFT but due to the finite observation time. Spectral Leakage gives rise to two problems : 1) The spectral component of the desired signal no longer contains the complete energy, rather it also contains the energy of adjacent components and noise, thereby reducing the Signal to noise ratio; 2) Spectral leakage from a larger signal component may significantly overshadow other smaller signals making them difficult to identify or detect.

Windowing Techniques are used to mitigate the effects of spectral leakage and therefore the restriction of having a limited observation interval.

Matlab Code:

Here is a matlab code to simulate the charts given in this discussion

```
%Illustration of Spectral Leakage and its effects on Frequency
Spectrum
%(FFT)
%Author: Mathuranathan Viswanathan
%http:gaussianwaves.blosgpot.com
%License - Creative Commons - cc-by-nc-sa

observationTime=1; %Input : Observation time interval change it to
100 and 1.4
%and see the effect on FFT
Fx=7; %Frequency of the sinusoid

Fs=100; %Sampling Frequency
t=0:1/Fs:observationTime;
x=1*sin(2*pi*Fx*t);
```

```
plot(t,x)
title('SineWave - Frequency = 7Hz');
xlabel('Times (s)');ylabel('Amplitude');

%Perform FFT
NFFX=2.^(ceil(log(length(x))/log(2)));
FFTX=fft(x,NFFX);%pad with zeros
NumUniquePts=ceil((NFFX+1)/2);
FFTX=FFTX(1:NumUniquePts);
MY=abs(FFTX);
MY=MY*2;
MY(1)=MY(1)/2;
MY(length(MY))=MY(length(MY))/2;
MY=MY/length(x);
f1=(0:NumUniquePts-1)*Fs/NFFX;

figure;
stem(f1,MY,'k');
title('FFT of the sine wave');
axis([0 50 0 1]);
xlabel('Frequency');ylabel('Amplitude');
```